Expanding and Factoring Patterns
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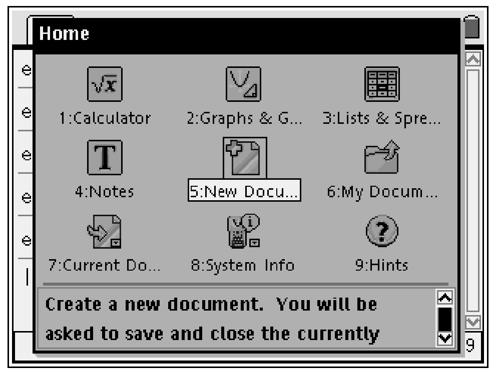
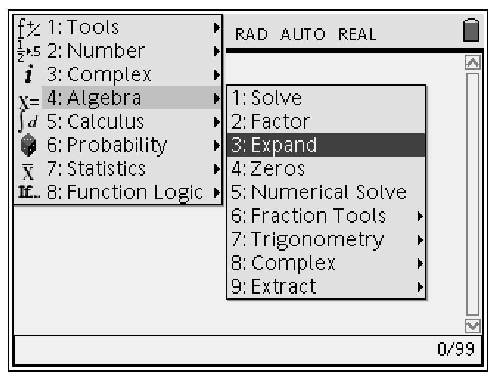
A CAS tool can be used to help students explore patterns in expanding and factoring. In fact, this is the one area of mathematics that many teachers cite as the topics that they are most hesitant to use a CAS for. Hopefully, this will help them see more than just the "black box" and see the idea of patterning to aid understanding.
Let's start with some simple expanding patterns. There are three that can be investigated easily – the difference of squares, the perfect square and simple trinomials. I've decided to begin with the difference of squares, but understand that other teachers would prefer a different order. I would also use algebra tiles wherever possible.

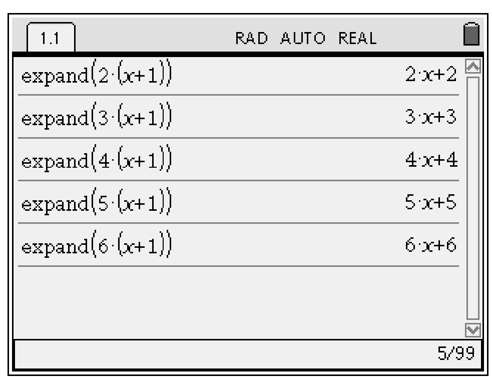
Note: The screens that you see here are from TI-*n*spire CAS, but could easily be done on the home application on the TI89 Titanium, the TI Voyage 200 or the TI 92 Plus.
To begin, start a new document press ⌂ and choose 5 for New Document. You might get a prompt asking if you wish to save the previous document. Once you have answered that prompt, press 1 to open a new Calculator page.



First, we'll need to find the expand command in the menu. Press menu for menu followed by 4 for Algebra and 3 for Expand. The command "expand()" will appear on the screen.



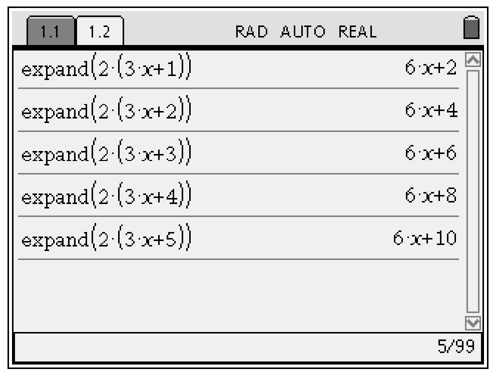
Complete the command to read "expand((x+1)(x-1)) and press enter. The device adds the multiplication symbol between the factors. The results are shown on the right of the screen. One of the best features of using a CAS is that your students can generate a large number of examples very quickly.
Given the five examples in the screen to the right, I would ask my class to answer two questions.
1) Can you explain the pattern in these examples? Explain patterns in both the questions and answers.
2) Can you predict what the results will be for the next two examples?



These two questions will be repeated throughout the development of skills.
The use of algebra tiles could be introduced at this juncture. I have found that the work with the two tools helps consolidate the students' skills.

By the way, the TI-*n*spire CAS device can do copy and paste in the same way that you would with Windows©. Move up to an expression that you want to copy. When it is highlighted, press (ctrl) followed by (C). This will copy the expression. Move to the next line and press (ctrl) followed by (V) to paste the expression. Then, move back to any values that you wish to change, press (clear) to delete the characters and type in the new values. If you need to select a particular bit of text, move to the front of the expression, press and hold the (CAPS) key while pressing ▶ on the Nav Pad (the donut). The device also allows others Windows© shortcuts that you may be familiar with.

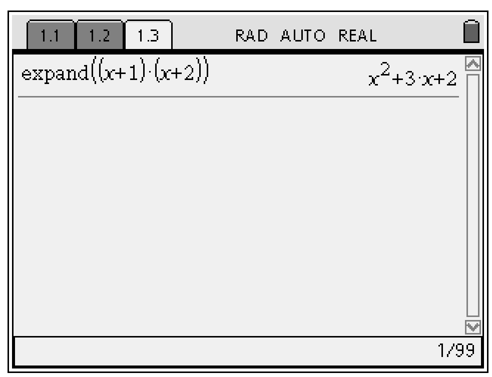
Next, we'll need to increase the difficulty level for the next set of problems. Press (home) followed by (1) for Calculator to start a new page.
This set begins with the same first problem as the previous set but changes the coefficient of the first term in each factor. Again, I'd want my class to answer the same two questions as before.



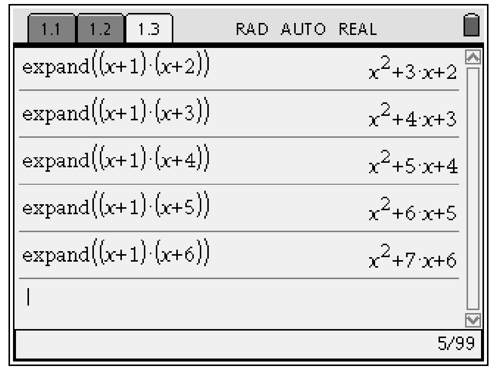
If I'm confident that my class has picked up the pattern, I could then assign a few examples for them to work on either independently or in pairs, without the CAS device.

Let's move on to simple trinomial expansions. Press (menu) followed by (1) to open a new page.
I'm sure that all teachers have tried the following set of questions:

1) How is the coefficient of 3 in the result related to the numbers in the factors?
2) How is the constant of 2 in the result related to the numbers in the factors?
3) Do you think that this pattern will work in another example?

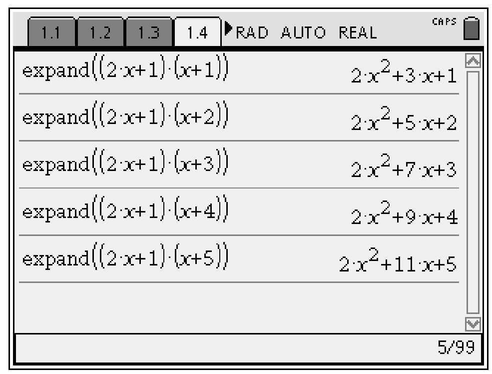


There’s only one way to find out – generate more examples. In this set, I have changed only one factor by increasing the value of the constant each time. I’m hoping that students will verify the pattern that they conjectured in the previous step, as well as answer the same two questions posed earlier. This set of examples provides a very good point to begin the use of algebra tiles if you haven’t done so already.



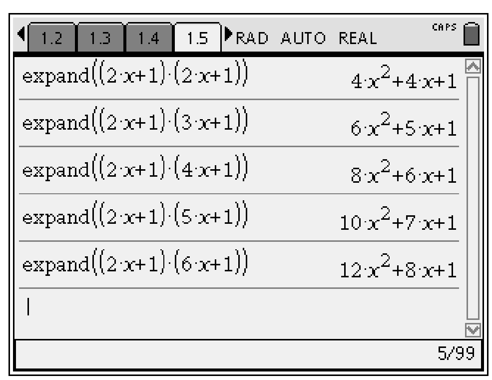
At this point, it would be wise to consolidate the pattern that students will have expressed – that the sum of the two constants in the factors produces the coefficient of the middle term and that the product of these values produces the constant in the result. Students will likely need some printed examples to attempt with the CAS device to anchor their knowledge. While algebra tiles can be used to support the development of these skills, I do not advise using algebra tiles with negative values.

If we increase the difficulty level one notch at a time, students have a much better chance of success in noticing patterns. In this screen, I have changed the coefficient of x to two in each of the first factors and allowed the constant in the second factor to increase by one in each problem. This is definitely one type of problem where the use of algebra tiles supports the development of concepts and skills.

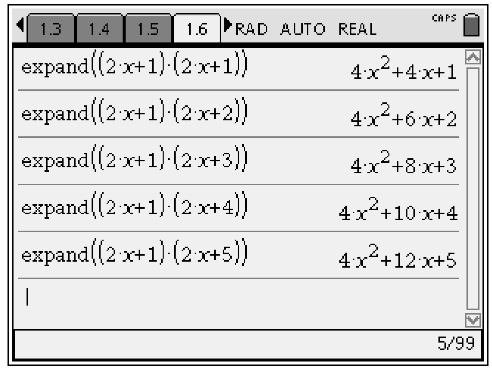


It would be easy for students to notice the patterns in the constants and the patterns in the linear terms and predict the results in the next two problems. There is always value in students observing such patterns, and this is not to be discouraged. However, what is needed is for them to expand the pattern observed above regarding the products and sums, incorporating the new factor of two.

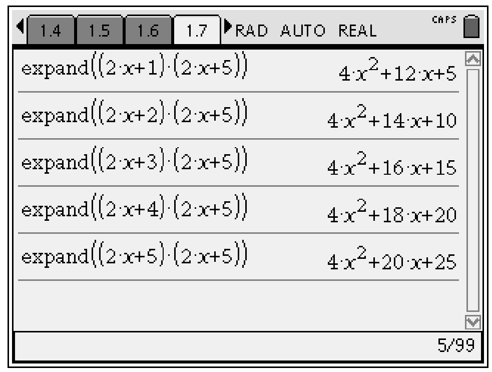
If we increase the coefficient of the variable in the second factor, we will have another pattern for students to detect, but more importantly, the results, along with a demonstration using algebra tiles, can consolidate the observations above.



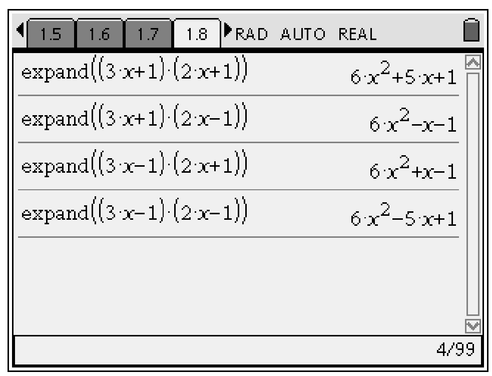
What remains is to generalize this result before moving on to special patterns. In this screen, the constant in the second factor increases by one with each new problem. If students have picked up the pattern above, they should be able to determine if these examples support and verify that pattern.



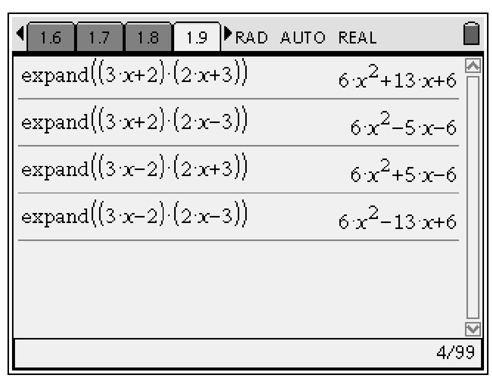
Finally, we vary the final occurrence of the unit value by increasing the constant in the first factor. While there are still linear patterns in the results, we need students to use these results to verify the product and sum observations made above.



What remains is to mix up positive and negative values to ensure that the pattern can be generalized even further. In this case, the constants have remained the same but the signs have been permuted to cover all possibilities.

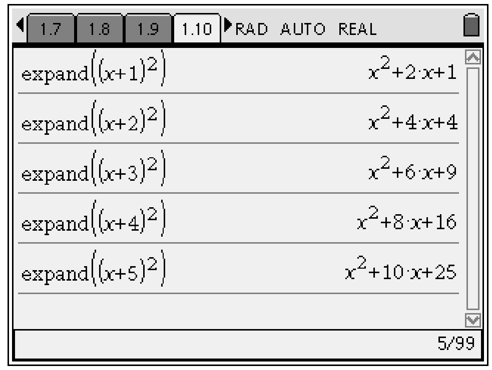


Repeating this pattern for another set of similar examples is again, quick and easy with a CAS device. There is the added bonus of some new patterns for students to notice in these examples.

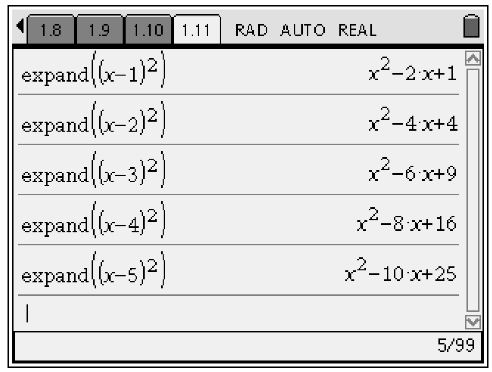


As before, a worksheet to provide practice with more generic problems would help students follow up on the development of skills and concepts.

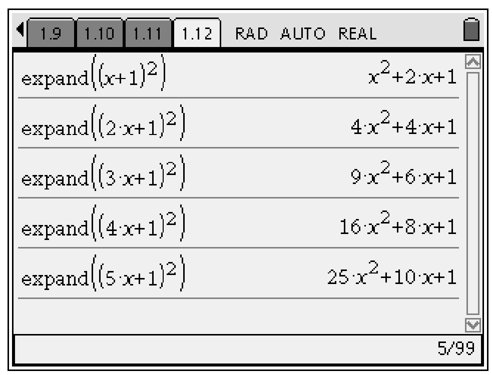
Let's move on to the perfect square expanding pattern. Press followed by 1 to open another new Calculator page.
We are hoping that students will see something similar in this pattern. This time, we also want them to pick up on the pattern in the linear term. As before, this set of examples provides an ideal point to use algebra tiles.



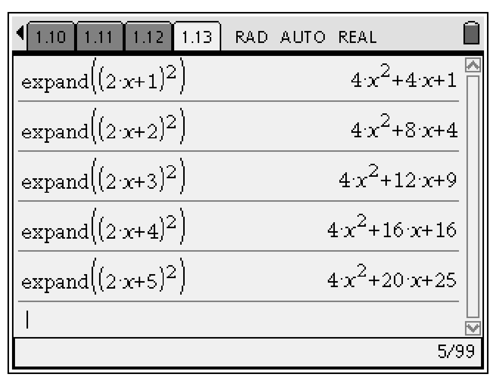
The same two questions should be answered, plus we want them to explore the same examples but replace the addition of terms with subtraction. The results are shown here, but the students should generate these on their own.



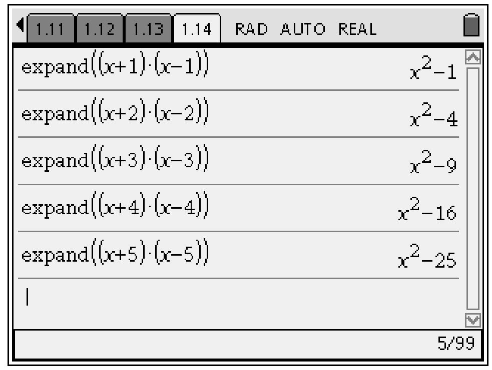
To increase the level of difficulty, we should introduce a coefficient with the variable. Press menu followed by 1 to open a new calculator page. The next set will resemble the previous examples, but this time, the coefficients are being changed on each line. Again, being able to generate examples quickly is a very convenient feature of a CAS. Students should pick up patterns and be able to answer the same two questions.



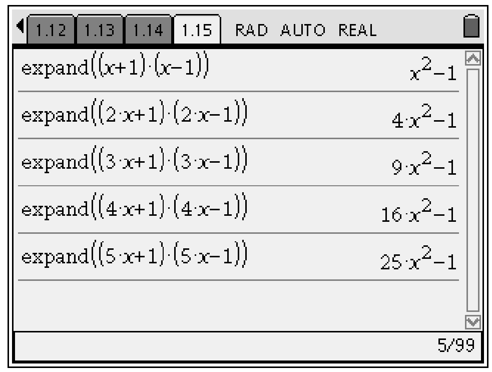
Before moving on to another pattern, extend this pattern one more time. Keeping the coefficient of the variable term constant and increasing the constant terms provides examples that extend the pattern.



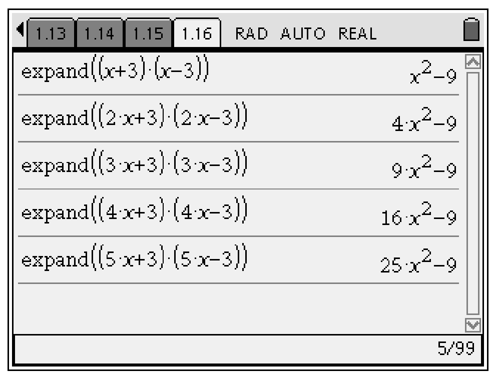
The final “special” pattern is the Difference of Squares. At this point, it is hoped that students’ experience with expanding the products of binomials would have provided sufficient experience to explain why each of these products is “missing” a middle or linear term.



To increase the level of difficulty, leave the constant term as 1 and vary the coefficient of the first term. It is hoped that students will comment upon the fact that these examples are very similar to those in the previous set.



To wrap this section up, we should attempt to show students more generic problems. Let’s fix the constant at 3 and vary the coefficient. In this way, they will have seen a variety of combinations.



As before, practice sheets to be done without the CAS will assist students with the development of skills.