White Box vs. Black Box

The Peel District School Board in Ontario has embarked on a journey in the use of CAS (Computer Algebra Systems) with a focus on grades nine and ten applied mathematics.

In the 2005-2006 school year, seven schools were provided with a class set of TI89 calculators and two days of training. In the 2006-2007 school year, 21 schools were given a class set of TI89s and three days of training. By the end of that school year, these TI89s were exchanged for class sets of Texas Instruments TI-Nspire.

So, what have they been learning? One of the key concepts that they have been looking at is two methods of using these devices – the white box approach and the black box approach. For this issue, I will look at solving linear equations in grade nine and systems of equations in grade ten for examples of the two approaches.

The black box approach would represent a use where the device is just a tool in the hands of the students. Refer to Figure 1. Students who are learning to solve equations would understand nothing about the method of solving equations if they were taught to use the “Solve” command on the devices. If our objective is to have students understand the algebra in solving equations, this method has failed the students badly.

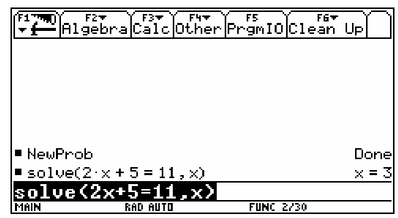

Figure 1

Contrast this with the white box approach. William G. McCallum of the University of Arizona defines a white box as “a transparent device that shows each step of the calculation, although the steps themselves are carried out by the machine”. (Editors – the reference here is taken from William G. McCallum, University of Arizona, Tuscon, Arizona, in “COMPUTER ALGEBRA SYSTEMS in secondary school mathematics”, ©2003, The National Council of Teachers of Mathematics, Reston, Virginia. ISBN 0-87353-531-6).

Using this approach, a student using the device will step through the solution using the same steps that we would expect using pencil-and-paper, with the device doing the mechanical steps. To begin, the student types in the equation and presses ENTER. If they understand the ideas behind solve equations, their next step would be to subtract 5 from each side. On the TI89 or TI92 devices, pressing − 5 displays as (2x+5=11)-5 with the result 2x=6 – see Figure 2. Teachers have a lot of problems with the form of the command on the left side of the screen. Students have fewer issues with this. The TI Nspire avoids this problem in its output.

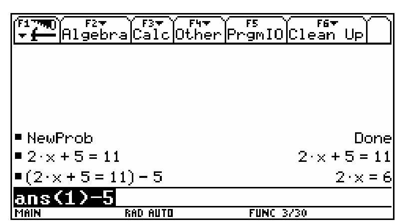

Figure 2

To complete the command, press ÷ 2 as shown in Figure 3. Using this approach, the device completes the calculations that the student initiates.

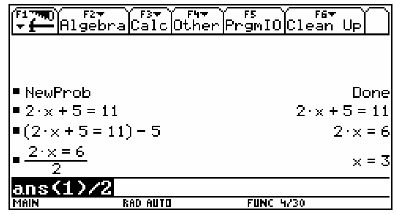

Figure 3

Another advantage of the white box approach is the option of allowing students to make the types of errors commonly done by novices and learn from those errors.

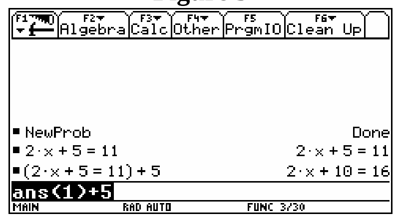

Figure 4

For the equation that we have been looking at, many students think that the numeric term "5" can be moved to the other side of the equation by adding 5 to both sides. So, let them try. The resulting equation is obviously incorrect as it is no simpler than the original equation. In fact, it is worse. From here, we can either have the student start over or take the correct step by subtracting 10 from each side in the next line.

CAS devices can also be used with algebra tiles to give a variety of solution methods. However, it must be noted that the objective remains to develop algebraic skills. I had a great deal of success with this approach and found the students were able to develop the skills they needed with the assistance of the CAS device and tiles.

For systems of equations, the black box approach involves a similar approach as for linear equations. The logical connector "and" is inserted between the two equations. The student presses ENTER and the solution is displayed – see Figure 5. However, the student understands absolutely nothing about solving a system of equations other than how to manipulate the device.

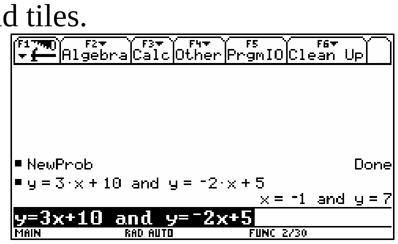

Figure 5

Let's look at a white box approach to this problem. This one is very easily done since the setup lends itself to the method of substitution. In the second equation, substitute 3x+10 for y to get the result "3x+10=-2x+5" – see Figure 6.

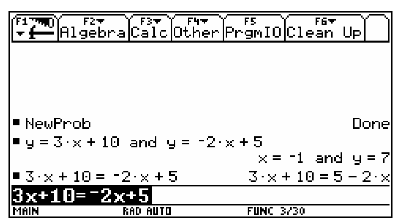

Figure 6

This is simply a linear equation and the student can use the device to solve for x in any way that the teacher prefers. In figure 7, the white box approach has been used to solve for x. Note the final command where the device has been used to substitute x=-1 into the first equation to get the value of y.

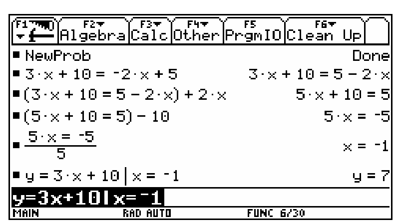

Figure 7

The system presented is rather simplistic, so let's look at something where the student has to isolate a variable as their first step. The system below adds a level of complexity that we need:

$3x - 4y - 6 = 0$

$2x + 3y + 13 = 0$

In Figure 8, the white box approach has been used to isolate y. Note that the choice of variable y is completely arbitrary in this example.

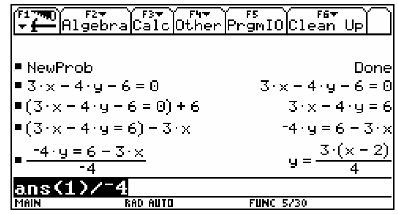

Figure 8

Once y has been expressed in terms of x, the resulting expression is substituted into the second equation in place of y. The result of this work is a linear equation in variable x. The device takes care of the expansion of the factors and simplification of the expression shown in

Figure 9.

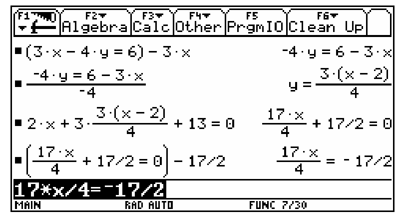

Figure 9

The work to solve this equation is shown in Figure 10. In the final command on this screen, the value of x has been substituted back into the original first equation.

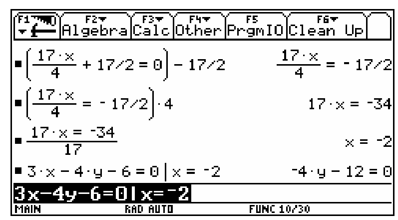

Figure 10

Finally, we solve for variable y as shown in Figure 11. Throughout this example, students will need to follow exactly the same sequence of steps that we would have expected them to do with pencil-and-paper.

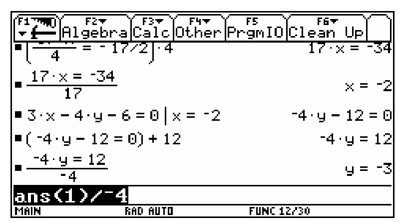

Figure 11

The "Solve" command is a dangerous command to show students. Once they have seen it, they will not want to go back to the process of stepping through the solution as we would like. If they use the black box approach with the systems of equations shown in figure 12, the answers will not make any sense to them. The white box approach lends itself better to these systems. I never showed the Solve command to my classes until after any assessments pertaining to this topic. However, once they have completed the basic work, I often gave them access to the CAS devices to solve equations using the Solve command.

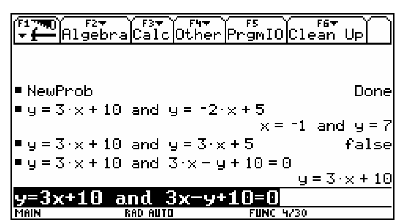

Figure 12

CAS devices are being used in many Ontario schools in several boards. I realize that there are many teachers who strenuously object to the use of these devices for fear that their students will only know how to use the devices. I have found the opposite, that teaching with these devices promotes understanding of algebraic properties.